

Department of Physics, Princeton University

**Graduate Preliminary Examination  
Part I**

Thursday, January 8, 2015  
9:00 am - 12:00 noon

Answer TWO out of the THREE questions in Section A (Mechanics) and TWO out of the THREE questions in Section B (Electricity and Magnetism).

Work each problem in a separate examination booklet. Be sure to label each booklet with your name, the section name, and the problem number.

## Section A. Mechanics

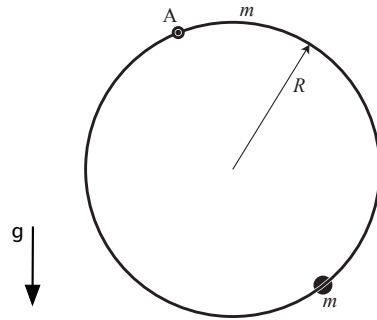
1. (Precession due the Sun's oblateness) In addition to the celebrated relativistic effect, the precession of the perihelion of Mercury can be affected by the deviation in the Sun's mass distribution from spherical symmetry, caused by Sun's rotation around its axis. The distortion creates a small correction to the gravitational potential, which along the plane perpendicular to the axis of rotation is:

$$\delta V(r) = -\lambda \frac{mMG}{r^n}$$

where  $G$  is the gravitational constant,  $M$  and  $m$  are the masses of the Sun and Mercury respectively,  $\lambda$  is a small (dimensionfull) parameter, and the power is  $n > 1$ .

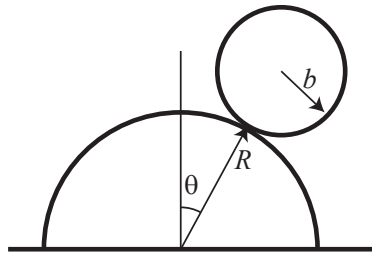
- (a) Calculate the orbital angular velocity,  $\omega_{\text{orb}}$ , for an orbit which is close to circular, at radius  $r$ , and lies within the plane perpendicular to the axis.
- (b) Write down the effective potential for radial motion and calculate the frequency of small oscillations about its minimum,  $\omega_{\text{osc}}$ , in terms  $r$  and the variables defined above.
- (c) From (a) and (b) find the approximate rate of precession of the perihelion (the point of closest approach) for a slightly elliptical orbit with an average radius  $r$ .
- (d) Depending on the sign of  $\lambda$  the precession may be in the same direction as the orbital angular velocity, or opposite to it. Which is it for  $\lambda > 0$ ?
- (e) What is the value of  $n$  and the sign of  $\lambda$  for the correction due to the Sun's mass quadrupole moment, if that is caused by oblateness (the sun's polar axis being shorter than its equatorial diameter)?

2. A thin hoop of mass  $m$  and radius  $R$  is suspended from its rim (point A) and is free to rotate around point A in the plane of the hoop. A small bead of equal mass  $m$  can slide without friction on the hoop.



Find the frequencies of the normal modes for this system.

3. A uniform cylinder of mass  $m$  and radius  $b$  rolls off a fixed cylindrical surface of radius  $R$  under the influence of gravity. The axes of both cylinders are horizontal. The rolling cylinder starts from the top of the fixed cylinder with a negligibly small velocity.



- (a) If we assume the cylinder rolls without slipping, find the angle  $\theta$  from the vertical when it loses contact with the fixed cylinder.
- (b) In practice for a finite value of  $\mu$  the cylinder will start to slip before it loses contact. Find the angle when it starts to slip for  $\mu = 1$ .

## Section B. Electricity and Magnetism

1. A small wire loop of radius  $a$  lies in the  $xy$ -plane, centered on the origin. A magnetic moment  $\mathbf{m} = m \hat{\mathbf{z}}$  travels up along the  $z$  axis with constant speed  $v$ . It passes through the center of the wire loop at  $t = 0$ .
  - (a) Compute the emf  $\mathcal{E}(t)$  around the loop.  
*Hint: the integral is easier if you evaluate the flux through a section of a spherical surface centered on the magnet and bounded by the wire loop rather than through the planar area bounded by the loop.*
  - (b) If the loop has resistance  $R$ , find the Joule heat  $P(t)$ . Assume the loop is fixed in position.
  - (c) Now consider the case where a uniform linear charge density  $\lambda$  is glued to a non-conducting loop (same orientation and radius as above), and the loop is allowed to spin. What is the position of  $\mathbf{m}$  at the time the loop attains its largest angular momentum,  $\mathbf{L}_{max}$ ? What is the value of  $\mathbf{L}_{max}$ ? Assume the dipole began its constant-velocity motion at  $t = -\infty$ , and that the loop was at rest then.

2. A Fresnel rhomb is an optical device used to convert linearly polarized light into circularly polarized light. As shown in Fig. 2, light hits the surface of the rhomb at normal incidence, it then undergoes two total internal reflections inside the rhomb, and then leaves the rhomb again normally.

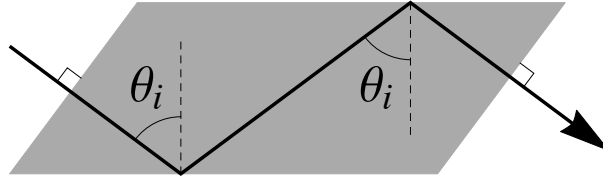


Figure 2: Fresnel rhomb.

The total internal reflections are such that each reflection generates a phase difference of  $45^\circ$  between the component of the light-wave that is parallel and the component that is perpendicular to the plane of incidence (the plane of the page in Fig. 2), and so after two internal reflections a lightwave that was originally linearly polarized at  $45^\circ$  with respect to the plane of incidence becomes circularly polarized.

- (a) For a single internal reflection, find the phase shift that the reflected wave acquires relative to the incident wave assuming the electromagnetic wave is polarized in the plane of incidence.

[Hint: Let  $r$  be the ratio between the complex amplitude of the reflected wave and that of the incident wave. At total internal reflection, one has  $|r| = 1$ , so  $r = e^{i\phi}$  for some  $\phi$ .]

- (b) Calculate the phase shift that the reflected wave acquires relative to the incident wave when the electromagnetic wave is polarized perpendicular to the plane of incidence.
- (c) If each of the two total internal reflections in a Fresnel rhomb occurs at an angle of incidence of  $\theta_i = 53.3^\circ$ , calculate the index of refraction  $n$  of the Fresnel rhomb relative to that of the surrounding medium.

[You might find useful the following trig. identity:  $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$ . ]

3. This problem explores some elements of a mass spectrometer. Parts (a) and (b) may be answered independently and treated non-relativistically.

- (a) An ion of charge  $+q$  and mass  $m$  is accelerated through a potential  $V_o$  as shown in Fig. 3(A). It enters a region between two very long (in the direction perpendicular to the page) cylindrical electrodes of radius  $a$  and  $b$  respectively. Find the potentials  $V(a)$  and  $V(b)$  such that the ion moves in a circle of radius  $r_o$ .

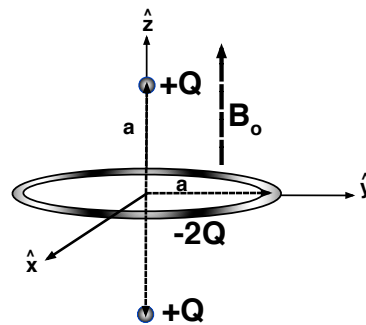
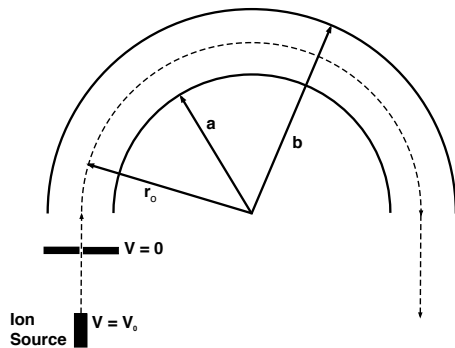


Figure 3: (A) Electrostatic filter.

(B) Penning trap.

- (b) A Penning trap is used in Fourier-transform mass spectrometry. At the simplest level, the Penning trap consists of a uniform magnetic field  $B_o \hat{z}$  and a quadrupole electric field. Let us assume that the electric field is generated by two positive charges  $+Q > 0$  located at  $\pm a \hat{z}$  and a uniformly charged ring of radius  $a$  and charge  $-2Q$  centered around the origin in the  $xy$ -plane, as shown in Fig. 3(B). This setup is rotationally symmetric around the  $z$ -axis.

- i. Close to the origin, the electric field takes the form

$$\mathbf{E} = k_z z \hat{z} + k_r r \hat{r},$$

where  $r = \sqrt{x^2 + y^2}$ . Determine  $k_z$  and  $k_r$ .

While the general motion of an ion of mass  $m$  and charge  $q > 0$  in the Penning trap is quite complicated, here we investigate only two particular cases:

- ii. Find the frequency  $\omega_z$  of small oscillations around the origin in the case where the ion moves only along the  $z$ -axis.
- iii. Assume the ion moves *uniformly* along a circle of radius  $R \ll a$  in the plane at  $z = 0$ . What is the angular frequency for this motion? Interpret the answer in the limit of large  $B_o$ .

Department of Physics, Princeton University

**Graduate Preliminary Examination  
Part II**

Friday, May 11, 2015  
9:00 am - 12:00 noon

Answer TWO out of the THREE questions in Section A (Quantum Mechanics) and TWO out of the THREE questions in Section B (Thermodynamics and Statistical Mechanics).

Work each problem in a separate booklet. Be sure to label each booklet with your name, the section name, and the problem number. Calculators are not permitted.



## Section A. Quantum Mechanics

1. Consider a toy model of the Helium atom where the Coulombic interaction potential is replaced with a Hooke's law potential. If the nucleus of the atom is located at  $\vec{r} = 0$  and the electrons of mass  $m$  have position vectors  $\vec{r}_1$  and  $\vec{r}_2$ , the interaction potential is

$$V(\vec{r}_1, \vec{r}_2) = \frac{1}{2}m\omega^2 (\vec{r}_1^2 + \vec{r}_2^2) - \frac{\lambda}{4}m\omega^2 (\vec{r}_1 - \vec{r}_2)^2 .$$

This model is exactly solvable. Assume  $\lambda > 0$ .

- (a) What constraint must be imposed on  $\lambda$  for the system to be well-behaved? [*Hint: It may be useful to consider the center of mass and relative position vectors of the two electrons  $\vec{u} = (\vec{r}_1 + \vec{r}_2)/2$  and  $\vec{v} = \vec{r}_1 - \vec{r}_2$ .*]
- (b) What are the energy levels of this system when  $\lambda = 1/2$ ?
- (c) Taking into account the spin of the electrons, what are the degeneracies of the lowest four energy levels when  $\lambda = 1/2$ ?
- (d) Suppose the Helium atom is initially in the third excited state. It then undergoes a decay through an electric dipole transition to a lower-energy state. What are the possible energies of the emitted photon?

2. A hydrogen atom located at  $\vec{r} = 0$  is initially in the  $2s$  state when an ion of charge  $Q$  passes by it. Assume the ion moves with constant velocity  $\vec{v} = v\hat{y}$  on a straight line whose closest approach to the hydrogen atom is  $\vec{b} = b\hat{z}$ , with  $b \gg a_B$ , where  $a_B$  is the Bohr radius. While the ion passes by, the electron in the atom experiences a time-dependent potential

$$V_1(\vec{r}, t) = \frac{Qe}{|\vec{b} + \vec{v}t - \vec{r}|}.$$

We are interested in calculating the transition probability to one of the  $2p$  states. In this problem, you may assume that the  $2s$  and  $2p$  states are degenerate.

- (a) Find an expansion of  $V_1(\vec{r}, t)$  that is valid at all times and that is appropriate for calculating the transition probability in the limit  $b \gg a_B$ . Identify the leading term in this expansion that will give a non-vanishing transition amplitude between the  $2s$  and at least one of the  $2p$  states in first-order time-dependent perturbation theory.
- (b) Using first-order time-dependent perturbation theory, calculate to leading order in  $a_B/b$  the probability that the atom winds up in a  $2p$  state.

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Some useful hydrogen atom wave-functions are:

$$\begin{aligned}\phi_{2s} &= \frac{1}{2\sqrt{2\pi a_B^3}} \left(1 - \frac{r}{2a_B}\right) e^{-r/(2a_B)}, \\ \phi_{2p,0} &= \frac{z}{4\sqrt{2\pi a_B^5}} e^{-r/(2a_B)}, \\ \phi_{2p,\pm 1} &= \frac{x \pm iy}{8\sqrt{\pi a_B^5}} e^{-r/(2a_B)}.\end{aligned}$$

You may also find the following integral useful:

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^{3/2}} = 2.$$

3. (a) Consider the Hamiltonian for a general time-independent, one-dimensional potential  $V(x)$ ,

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x).$$

Show that for an arbitrary, continuous function  $\phi(x)$ , the value of

$$E = \frac{\langle \phi | H | \phi \rangle}{\langle \phi | \phi \rangle}$$

gives an upper bound on the ground state energy for the potential  $V(x)$ .

- (b) For a particle moving in a triangular potential well

$$V(x) = \begin{cases} \infty & \text{if } x < 0, \\ V_0 x/L & \text{if } x > 0, \end{cases}$$

the energy levels take the form

$$E_n = \alpha_n V_0 \left( \frac{\hbar^2}{mL^2 V_0} \right)^q,$$

where  $\alpha_n$  and  $q$  are numerical constants. Determine the value of the exponent  $q$ .

- (c) Using the approach in part (a), find an estimate for the constant  $\alpha_0$  corresponding to the ground state in the triangular potential well. (The estimate needs not be optimal, but should be based on a reasonable variational calculation.)

## Section B. Statistical Mechanics and Thermodynamics

1. (**The weather underground**) Assume the Earth is flat with its surface at  $z = 0$ . The solid material below the surface has a temperature-independent thermal diffusivity  $D$ . The weather above ground is a highly regular climate with sinusoidal annual (a) and daily (d) oscillations of the temperature, so the temperature at the surface as a function of time  $t$  is

$$T(t, z = 0) = T_o + T_a \cos(\omega_a t) + T_d \cos(\omega_d t) .$$

Assume the temperature at infinite depth below ground ( $z \rightarrow -\infty$ ) is  $T_o$ .

- (a) What is the temperature  $T(t, z)$  below ground at time  $t$  and position  $z < 0$ ?
- (b) What is the position  $z$  closest to the surface where the annual temperature cycle is opposite to that at the surface (so that below Princeton it is instead “hottest” in January and “coldest” in July)?
- (c) By what factor is the annual temperature variation attenuated at the above depth?

**2. (Atmospheric density)**

- (a) If you hold up your hand, how many molecules per second are colliding with your palm? We are interested in the order of magnitude, not the last factor of two. Approximate the atmosphere as pure oxygen (of molecular mass 32 a.m.u.).

Suppose that the atmosphere was in equilibrium at constant temperature. Then, in addition to the kinetic energy of the molecules in the air, one would have to take account of the potential energy due to gravity. Under these assumptions:

- (b) What is the probability distribution for the height  $h$  of a molecule above the surface of the earth?
- (c) How much less oxygen would you find at a height of 1 km than you do at the earth's surface?

Some relevant constants:  $R \approx 8.3 \frac{\text{J}}{\text{mol K}}$ ,  $N_A \approx 6.02 \times 10^{23} \text{mol}^{-1}$ .

**3. (An ideal Bose gas)**

Consider an ideal Bose gas of particles with zero spin and the dispersion relation

$$E_{\vec{k}} = c |\vec{k}|.$$

- (a) Show that at high temperatures the mean energy per particle for this gas satisfies

$$U/N = \alpha T$$

with a constant  $\alpha$ , and determine the value of that constant (in three dimensions).

Hint: a useful relation is  $\int_0^\infty dx x^\nu \beta e^{-x\beta} = \nu \int_0^\infty dx x^{\nu-1} e^{-x\beta}$  (for  $\beta, \nu > 0$ ).

- (b) The pressure of such a gas with  $N$  particles becomes independent of  $N$  below a critical temperature,  $T_c$ . Explain why and calculate this temperature.
- (c) Calculate the specific heat  $c_V$  of the gas for  $T < T_c$ .
- (d) Does the transition occur if such a gas is confined to two spacial dimensions?

Department of Physics, Princeton University

**Graduate Preliminary Examination**  
**Part I**

Thursday, May 7, 2015  
9:00 am - 12:00 noon

Answer TWO out of the THREE questions in Section A (Mechanics) and TWO out of the THREE questions in Section B (Electricity and Magnetism).

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## Section A. Mechanics

### Rolling Sphere

1. A cylindrical pipe of interior radius  $R$  lies horizontally on the ground and remains stationary. A solid sphere of radius  $r$  is placed inside the pipe. What is the frequency of small oscillations of the sphere? Assume that the axis of rotation of the sphere is always parallel to the symmetry axis of the pipe, and the sphere rolls without slipping.



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2. A particle of mass  $m$  moves under the influence of an attractive central force with potential  $V(r)$ .
- (a) Suppose the particle to move in a circular orbit with angular momentum  $\ell$ . Derive the condition that determines how the orbit radius  $r_\ell$  depends on  $\ell$ .
  - (b) Now consider a small perturbation  $\delta r(t)$  around such a circular orbit. What is the condition that must be met for this perturbation to oscillate with a real frequency  $\omega_\ell$  (in which case the perturbation will not grow with time and the circular orbit will be stable)?
  - (c) Now consider the special case  $V(r) = -k/r^n$  with  $n > 0$ . For what values of  $n$  and  $\ell$  are circular orbits stable (i.e. such that  $\omega_\ell^2 > 0$ )?
  - (d) Are there any circumstances where the period of the circular orbit matches the period of small radial oscillations about the circular orbit. If so, what does this equality imply for the long-time trajectory of the particle?

**Unicycle**

3. One of the challenges of Jadwin Hall you no doubt have experienced is riding a unicycle down the ramp and not falling over as you do the  $180^\circ$  turn midway down the ramp. In this simplified study of unicycle stability, we assume that the total mass of the wheel plus rider is  $M$ , we assume the moment of inertia of the rider is zero, that the wheel has mass  $m$ , radius  $\ell$ , and moment of inertia  $I_w = m\ell^2$  about its axis, and that the center of mass  $H_{cm}$  of the unicycle and rider is at the top of the wheel. As you negotiate the turn, the unicycle moves with velocity  $V$  in a circular path of radius of  $R \gg \ell$ .

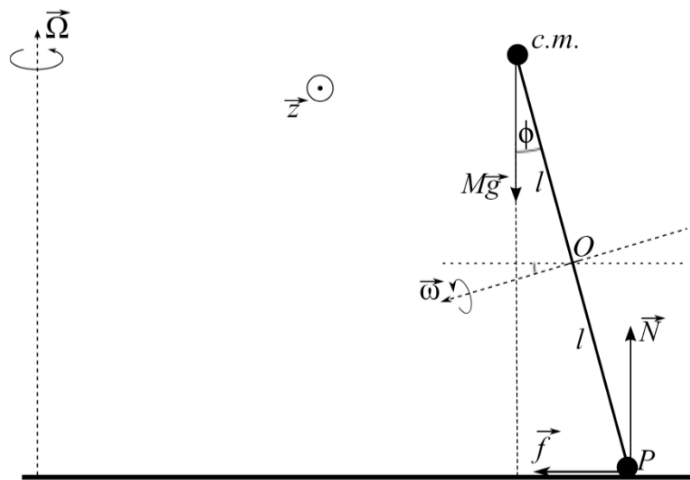


Figure 1: Coordinate system for the problem.

- (a) Find the angle  $\phi$  that the unicycle must lean through in the turn.
- (b) Put in some typical numbers and discuss the role of the angular momentum of the wheel in the stability of the unicycle. You can probably do this part successfully without the correct answer to (a).

## Section B. Electricity and Magnetism

### Radiating charge

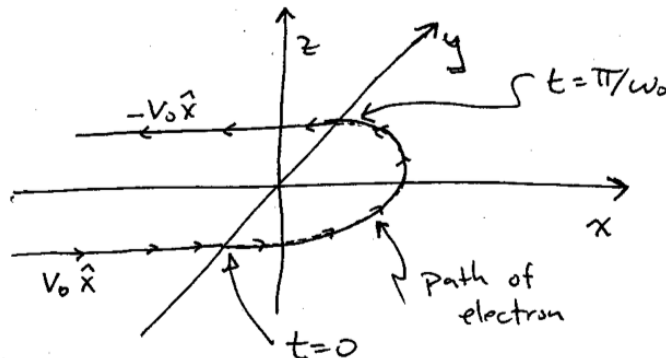
1. A magnetic field  $\mathbf{B}(x)$  points in the  $\hat{z}$  direction in half of space as follows.

$$\mathbf{B}(x) = 0 \quad \text{for } x < 0 \quad (1)$$

$$\mathbf{B}(x) = B_0 \hat{z} \quad \text{for } x > 0 \quad (2)$$

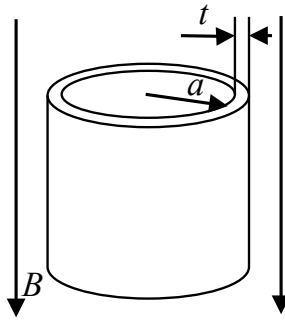
There is no electrostatic field. A non-relativistic electron of mass  $m_e$  initially moves at constant velocity  $\mathbf{v} = v_0 \hat{x}$  at  $y = -a$  in the  $xy$  plane in the space with  $x < 0$ . As shown in the figure, at  $t = 0$  the electron enters the space  $x > 0$ , travels in a semi-circle of radius  $a$  with an angular frequency of  $\omega_0 = eB_0/m_e$  for a time  $\pi/\omega_0$  and then exits the region of field with  $\mathbf{v} = -v_0 \hat{x}$  at  $y = a$ .

- (a) What is the differential power per unit solid angle,  $dP(t)/d\Omega$ , that an observer on the  $x$ -axis at a distance  $x = r\hat{x} \gg c/\omega_0$  measures when observing the charge? Sketch  $dP(t)/d\Omega$  as a function of time and quantitatively label the duration, amplitude, and key features. [Hint: A lot of progress can be made using a simple model of how a charge radiates.]
- (b) For an observer on the  $z$ -axis at a distance  $z = r\hat{z} \gg c/\omega_0$ , again sketch the  $dP(t)/d\Omega$  he or she would measure as a function of time. Quantitatively label the duration, amplitude, and key features.
- (c) What is the total energy emitted by the electron?



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2. A thin sheet of material with  $\epsilon_r = 1$  lies in the  $xy$  plane and carries a periodic surface charge density given by  $\sigma(x) = \sigma_0 \cos(kx)$ .
- (a) What is the electric field in the region  $z > 0$ ? [Hint:  $\nabla^2 V = 0$  in free space.]
- (b) A small neutral object of polarizability  $\alpha$  is placed at  $\mathbf{r} = (0, 0, z_0)$ . What is the force (magnitude and direction) on it?

3. Consider shielding of a uniform oscillating magnetic field  $B = B_0 \cos(\omega t)$  by a long open metal cylinder with its axis parallel to the field. The cylinder has electrical conductivity  $\sigma$ , radius  $a$ , height  $h$ , and wall thickness  $t$ , with  $t \ll a$ . Find the amplitude of the oscillating magnetic field inside the cylinder. Ignore edge effects and assume  $\omega \ll h/c$ .



Department of Physics, Princeton University

**Graduate Preliminary Examination  
Part II**

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9:00 am - 12:00 noon

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## Section A. Quantum Mechanics

1. A particle of mass  $m$ , is moving on a line under the action of the Hamiltonian:

$$H_L = \frac{1}{2m}p^2 - b^2 [\delta(x + L) + \delta(x - L)]$$

(whose potential features a pair of attracting delta functions).

- (a) State the symmetry, and sketch the shapes of the ground state and of the first excited bound state, in case such a state exists.
- (b) Determine the minimal  $L_0$  such that for  $L > L_0$  the Hamiltonian has more than one bound state.
- (c) What is the maximal number of bound states that  $H_L$  can have?

2. A point particle of mass  $m$  and electric charge  $q$  moves in a 3d harmonic oscillator potential with frequency  $\omega$  and a uniform electric field of strength  $\mathcal{E}$  pointing in the  $z$ -direction. The Hamiltonian is

$$H = \frac{\vec{p}^2}{2m} + \frac{m\omega^2\vec{r}^2}{2} - q\mathcal{E}z.$$

- (a) What are the eigenenergies of this Hamiltonian?  
(b) Find an expression for the ground state wave-function.

Assume now that the system is described by the above Hamiltonian only for  $t < 0$ , and that at  $t = 0$  the electric field is suddenly turned off. For  $t < 0$ , the system is in its ground state.

- (c) What is the probability that the system will end up in the new ground state right after the electric field is turned off?  
(d) What is the expectation value of the electric dipole moment  $\vec{d} = q\vec{r}$  at some given time  $t > 0$ ?



3. Consider the scattering of a spinless particle of charge  $e$  and mass  $m$  in the screened Coulomb potential (also known as the Yukawa potential),

$$V(r) = -\frac{Ze^2}{r}e^{-r/a}.$$

Compute the differential scattering cross section,  $d\sigma/d\Omega$ , for fast incident particles in the appropriate approximation. What is the range of validity for your approximation?

## Section B. Statistical Mechanics and Thermodynamics

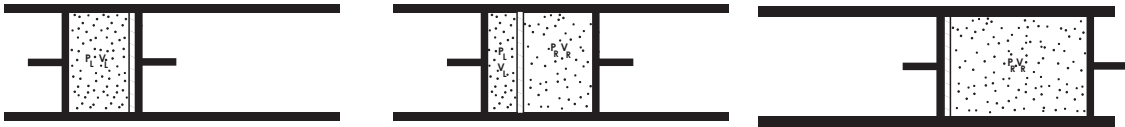
1. Consider a system of noninteracting spin trimers. Each trimer is a set of three spins each of which can be either “up” or “down.” Each trimer is described by the Hamiltonian

$$\hat{H} = -J(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) - \mu_0 H(\sigma_1 + \sigma_2 + \sigma_3) . \quad (1)$$

Here  $J$  is the spin coupling energy and  $H$  is an external magnetic field. The individual spin polarizations  $\sigma_i$  are two-state Ising variables, with  $\sigma_i = \pm 1$ .

- (a) Find the single trimer partition function  $Z_1$ .
- (b) Find the magnetization per trimer  $m = \mu_0 \langle \sigma_1 + \sigma_2 + \sigma_3 \rangle$ .
- (c) Suppose there are  $N_\Delta$  trimers in a volume  $V$ . The magnetization density is  $M = N_\Delta m/V$ . Find the zero field susceptibility [Hint: the definition of susceptibility is  $\chi(T) = (\partial M/\partial H)_{H=0}$ .]
- (d) Find the entropy  $S(T, H, N_\Delta)$ .
- (e) In the three limits  $J \rightarrow +\infty$ ,  $J \rightarrow 0$ , and  $J \rightarrow -\infty$  the expression for the entropy simplifies considerably. Give a simple physical explanation for each of these simplifications.

2. The figure below shows a throttling process in which two pistons “push” and “pull” gas through a porous divider (shown as a lightly hatched line) that is fixed inside a thermally insulated cylinder. No heat flows into or out of the cylinder. Initially, all the gas is on the left hand side of the divider and the right hand piston is up against the divider (top left plot). The top right figure shows an intermediate state and the bottom figure shows the final state. The pistons are moved in such a way that the pressure on the left hand side is always  $P_L$  and the pressure on the right side is  $P_R$ , with  $P_R < P_L$ . The final volume is larger than the initial volume.



- (a) What thermodynamic potential has the same value at the end of the process as it did at the start? Prove it!
- (b) If the gas is ideal, what is its change in the internal energy between initial and final states?
- (c) Suppose now that the gas has a van der Waals equation of state

$$\left(P + \frac{N^2 a}{V^2}\right)(V - Nb) = NkT \quad (2)$$

where  $a$  and  $b$  are small numbers. The Helmholtz free energy is

$$F = -NkT \left\{ \ln[n_Q(V - Nb)/N] + 1 \right\} - N^2 a/V \quad (3)$$

where  $n_Q = (mkT/2\pi\hbar^2)^{3/2}$ . What is the internal energy  $U$ ?

- (d) What is the condition on the van der Waals coefficients  $a$  and  $b$  such that the gas cools on expansion into a very large volume in a throttling process? Qualitatively interpret the result. [Hint:  $0 < Nb/V \ll 1$ ]

### Adiabatic demagnetization

3. Consider a classical ideal gas of identical non-interacting particles of mass  $m$  in a container of volume  $V$  at initial temperature  $T_i$ . Let the particles have spin one-half and magnetic moment  $\mu$ , and let the container be placed in a strong magnetic field  $H$ .
- (a) Compute the classical partition function for this system, taking proper account of particle identity. It may help you to know that the partition function for a single classical particle (ignoring the spin degree of freedom) is  $Z_1 = n_Q V$  where  $n_Q = (mkT/2\pi\hbar^2)^{3/2}$ .
  - (b) Calculate the total energy and entropy for this system
  - (c) Now suppose that the container is thermally isolated and that the magnetic field is slowly reduced (i.e. adiabatically). Show that the temperature decreases continuously as  $H$  is decreased.
  - (d) Show that if  $H$  is reduced all the way to zero the final temperature satisfies the inequality  $T_i > T_f > 2^{-\frac{2}{3}} T_i$ .